CHAPTER 10

INTRODUCTION TO TRIGONOMETRY & TRIGONOMETRIC IDENTITIES

Syllabus

- Introduction to Trigonometry : Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined) motivate the ratios, which are defined at 0° and 90°. Values of the trigonometric ratios of 30°, 45° and 60°. Relationships between the ratios.
- Trigonometric Identities : Proof and applications of the identity, $\sin^2 A + \cos^2 A = 1$. Only simple identities to be given. Trigonometric ratios of complementary angles.

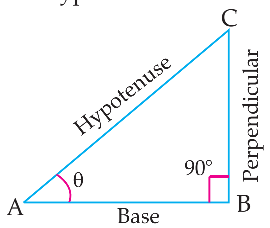
Trend Analysis

List of Concepts	2018		2019		2020	
	Delhi	Outside Delhi	Delhi	Outside Delhi	Delhi	Outside Delhi
Trigonometric Ratios and Complementary Angles	3 Q (1 M) 1 Q (3 M)		1 Q (1 M)	1 Q (1 M) 1 Q (3 M)	3 Q (1 M) 1 Q (2 M)	3 Q (1 M)
Trigonometric Identities	1 Q (4 M)		2 Q (3 M) 2 Q (4 M)	1 Q (4 M)	2 Q (1 M) 3 Q (3 M)	3 Q (1 M) 2 Q (2 M) 4 Q (3 M)

TOPIC - 1
Trigonometric Ratios and Complementary Angles

Revision Notes

- In fig., a right triangle ABC right angled at B is given and $\angle BAC = \theta$ is an acute angle. Here side AB which is adjacent to $\angle A$ is base, side BC opposite to $\angle A$ is perpendicular and the side AC is hypotenuse which is opposite to the right angle B.



Know the Formulae

The trigonometric ratios of ∠A in right triangle ABC are defined as

$$\text{sine of } \angle A = \sin\theta = \frac{\text{Perpendicular or opposite side}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

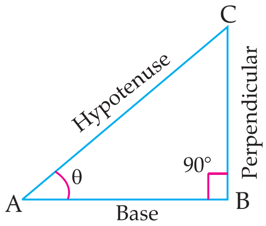
$$\text{cosine of } \angle A = \cos\theta = \frac{\text{Base or adjecent side}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\text{tangent of } \angle A = \tan\theta = \frac{\text{Perpendicular or opposite side}}{\text{Base adjacent side}} = \frac{BC}{AB}$$

$$\text{cotangent of } \angle A = \cot\theta = \frac{\text{Base or adjacent side}}{\text{Perpendicular or oppsite side}} = \frac{AB}{BC} = \frac{1}{\tan\theta}$$

$$\text{secant of } \angle A = \sec\theta = \frac{\text{Hypotenuse}}{\text{Base or adjacent side}} = \frac{AC}{AB} = \frac{1}{\cos\theta}$$

$$\text{cosecant of } \angle A = \operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular or opposite side}} = \frac{AC}{BC} = \frac{1}{\sin\theta}$$

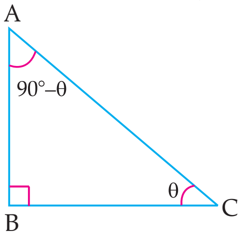


It is clear from the above ratios that cosecant, secant and cotangent are the reciprocals of sine, cosine and tangent respectively.

Also, $$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

and $$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

- The trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and length of its sides.
- The value of trigonometric ratio of an angle does not depend on the size of the triangle but depends on the angle only.
- **Complementary Angles:**
  Two angles are said to be complementary if their sum is 90°. Thus, (in fig.) ∠A and ∠C are complementary angles.



- **Trigonometric Ratios of Complementary Angles:**
  We have, BC = Base, AB = Perpendicular, and AC = Hypotenuse, with respect to θ.

∴ $$\sin\theta = \frac{AB}{AC}, \cos\theta = \frac{BC}{AC}, \tan\theta = \frac{AB}{BC}$$

and $$\operatorname{cosec}\theta = \frac{AC}{AB}, \sec\theta = \frac{AC}{BC}, \cot\theta = \frac{BC}{AB}.$$

Again, with respect to the angle (90° – θ), BC= Perpendicular, AB = Base and AC = Hypotenuse

∴ $$\sin(90° - \theta) = \frac{BC}{AC} = \cos\theta$$

$$\cos(90° - \theta) = \frac{AB}{AC} = \sin\theta$$

$$\tan(90° - \theta) = \frac{BC}{AB} = \cot\theta$$

$$\cot(90° - \theta) = \frac{AB}{BC} = \tan\theta$$

$$\sec(90° - \theta) = \frac{AC}{AB} = \text{cosec}\,\theta$$

$$\text{cosec}(90° - \theta) = \frac{AC}{BC} = \sec\theta$$

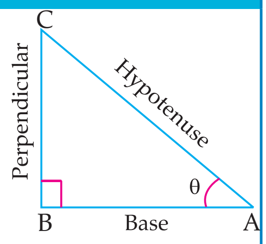
∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined (∞)
cot A	Not defined (∞)	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined (∞)
cosec A	Not defined (∞)	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Mnemonics

Concept

The relation of Trigonometric Ratios

In right angled ΔABC, we have



$\sin\theta = \frac{BC}{AC}$, $\cos\theta = \frac{BA}{AC}$, $\tan\theta = \frac{BC}{AB}$,

$\cot\theta = \frac{AB}{BC}$, $\sec\theta = \frac{AC}{BA}$, $\text{cosec}\,\theta = \frac{AC}{BC}$

sin	cos	tan
Pandit	Badri	Prasad
Har	Har	Bhole
cosec	sec	cot

Interpretation:

Here,

$$\sin\theta = \frac{\textbf{P}\text{andit}}{\textbf{H}\text{ar}} = \frac{P}{H} = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\cos\theta = \frac{\textbf{B}\text{adri}}{\textbf{H}\text{ar}} = \frac{B}{H} = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BA}{AC}$$

$$\tan\theta = \frac{\textbf{P}\text{rasad}}{\textbf{B}\text{hole}} = \frac{P}{B} = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$

$$\cot\theta = \frac{\textbf{B}\text{hole}}{\textbf{P}\text{rasad}} = \frac{B}{P} = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC}$$

$$\sec\theta = \frac{\textbf{H}\text{ar}}{\textbf{B}\text{adri}} = \frac{H}{B} = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{BA}$$

$$\text{cosec}\,\theta = \frac{\textbf{H}\text{ar}}{\textbf{P}\text{andit}} = \frac{H}{P} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{BC}.$$

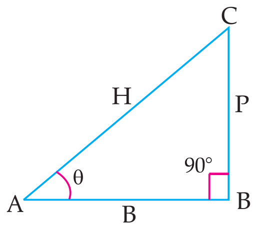
Trigonometric Ratios

Hints: We learn these ratios in following ways:

"**S**ome **p**eople **h**ave" $\sin\theta = \frac{P}{H}$

"**C**urly **B**rown **H**air" $\cos\theta = \frac{B}{H}$

"**T**hrough **p**roper **B**rushing" $\tan\theta = \frac{P}{B}$.



(i) $\sin\theta = \frac{BC}{AC} = \frac{P}{H}$

Interpretation:

Some	**P**eople	**H**ave
↓	↓	↓
$\sin\theta$	Perpendicular	Hypotenuse

(ii) $\text{Cos}\,\theta = \frac{AB}{AC} = \frac{B}{H}$

Interpretation:

Curly	**B**rown	**H**air
↓	↓	↓
$\cos\theta$	Base	Hypotenuse

(ii) $\tan\theta = \frac{BC}{AB} = \frac{P}{B}$

Interpretation:

Through	**P**roper	**B**rushing
↓	↓	↓
$\tan\theta$	Perpendicular	Base

How is it done on the GREENBOARD?

Q.1. If $\sqrt{2}\sin\theta = 1$, find the value of $\sec^2\theta - \text{cosec}^2\theta$.

Solution

Step I: Given $\sqrt{2}\sin\theta = 1$

or, $\sin\theta = \frac{1}{\sqrt{2}} = \sin 45^\circ$ ...(i)

Step II: $\theta = 45^\circ$ ...(ii)

Step I: Now, $\sec^2\theta - \text{cosec}^2\theta$.

$= (\sec^2 45^\circ)^2 - (\text{cosec}^2 45^\circ)^2$

$= (\sqrt{2})^2 - (\sqrt{2})^2$

$= 2 - 2$

$= 0$

Very Short Answer Type Questions — 1 mark each

AI **Q. 1. If sin A + cos B = 1, A = 30° and B is an acute angle, then find the value of B.**

R [CBSE SQP, 2020-21]

Sol. $\sin 30° + \cos B = 1$

$\frac{1}{2} + \cos B = 1$ ½

$\therefore \quad \cos B = 1 - \frac{1}{2} = \frac{1}{2}$

i.e., $\cos B = \cos 60° \quad \left[\because \cos 60° = \frac{1}{2}\right]$

Hence, $\angle B = 60°.$ ½

[CBSE Marking Scheme, 2020-21]

Q. 2. Find the value of $\frac{\cos 80°}{\sin 10°} + \cos 59° \operatorname{cosec} 31°$.

R [CBSE Delhi Set-I, 2020]

Sol. $\frac{\cos 80°}{\sin 10°} + \cos 59° \operatorname{cosec} 31°$

$= \frac{\cos(90° - 10°)}{\sin 10°} + \cos(90° - 31°) \times \frac{1}{\sin 31°}$ ½

$= \frac{\sin 10°}{\sin 10°} + \sin 31° \times \frac{1}{\sin 31°}$

$[\because \cos(90° - \theta) = \sin\theta \text{ and } \operatorname{cosec}\theta = \frac{1}{\sin\theta}]$

$= 1 + 1 = 2.$ ½

Q. 3. Find the value of $\left(\frac{\sin 35°}{\cos 55°}\right)^2 + \left(\frac{\cos 43°}{\sin 47°}\right)^2 - 2\cos 60°$.

R [CBSE Delhi Set-II, 2020]

Sol. $\left(\frac{\sin 35°}{\cos 55°}\right)^2 + \left(\frac{\cos 43°}{\sin 47°}\right) - 2\cos 60°$

$= \left[\frac{\sin(90° - 55°)}{\cos 55°}\right]^2 + \left[\frac{\cos(90° - 47°)}{\sin 47°}\right] - 2\cos 60°$

$= \left(\frac{\cos 55°}{\cos 55°}\right)^2 + \left(\frac{\sin 47°}{\sin 47°}\right)^2 - 2\cos 60°$ ½

$[\because \sin(90° - \theta) = \cos\theta \text{ and } \cos(90° - \theta = \sin\theta)]$

$= (1)^2 + (1)^2 - 2 \times \frac{1}{2} \quad \left[\because \cos 60° = \frac{1}{2}\right]$

$= 1 + 1 - 1 = 1.$ ½

Q. 4. Find the value of $\frac{2\cos 67°}{\sin 23°} - \frac{\tan 40°}{\cot 50°} - \cos 0°$.

R [CBSE Delhi Set-III, 2020]

Sol. $\frac{2\cos 67°}{\sin 23°} - \frac{\tan 40°}{\cot 50°} - \cos 0°$

$= \frac{2\cos(90° - 23°)}{\sin 23°} - \frac{\tan(90° - 50°)}{\cot 50°} - \cos 0°$

$[\because \cos(90° - \theta) = \sin\theta \text{ and } \tan(90° - \theta) = \cot\theta]$ ½

$= \frac{2\sin 23°}{\sin 23°} - \frac{\cot 50°}{\cot 50°} - \cos 0°$

$= 2 - 1 - 1 \quad [\because \cos 0° = 1]$

$= 0.$ ½

AI **Q. 5. Find the value of (tan 1° tan 2° tan 3°... tan 89°).**

U [CBSE OD Delhi Set-I, 2020]

Sol. (tan 1° tan 2° tan 3° ... tan 89°)

= (tan 1° tan 89°)(tan 2° tan 88°)(tan 3° tan 87°) (tan 44° tan 46°) ... (tan 45°)

= [tan 1° tan (90° – 1)][tan 2° tan (90° – 2)] [tan 3° tan (90° – 3)] ... [tan 45° tan (90° – 45°)]

= tan 1° cot 1° tan 2° cot 2° tan 3° cot 3° (tan 44° cot 44°) ... tan 45° ½

$= \tan 1° \times \frac{1}{\tan 1°} \tan 2° . \frac{1}{\tan 2°} \tan 3° . \frac{1}{\tan 3°} \ldots \frac{\tan 44°}{\tan 44°} \tan 45°$

= 1.1.1.1....1.1

= 1. ½

Q. 6. If tan A = cot B, then find the value of (A + B).

R + U [CBSE OD Set-II, 2020]

Sol. $\tan A = \cot B$ (Given)

$\Rightarrow \tan A = \tan(90° - B)$

$[\because \tan(90° - \theta) = \cot\theta]$

$\Rightarrow A = 90° - B$

Hence, $A + B = 90°.$ 1

Q. 7. Find the value of $\frac{\tan 35°}{\cot 55°} + \frac{\cot 78°}{\tan 12°}$.

R [CBSE OD Set-III, 2020]

Sol. $\frac{\tan 35°}{\cot 55°} + \frac{\cot 78°}{\tan 12°} = \frac{\tan(90° - 55°)}{\cot 55°} + \frac{\cot(90° - 12°)}{\tan 12°}$

$= \frac{\cot 55°}{\cot 55°} + \frac{\tan 12°}{\tan 12°}$

$[\tan(90° - \theta) = \cot\theta \text{ and } \cot(90° - \theta) = \tan\theta]$ ½

$= 1 + 1 = 2.$ ½

Q. 8. If sin α = $\frac{\sqrt{3}}{2}$ and cos β = 0, then find the value of β – α.

U [CBSE SQP, 2020]

Sol. 30°

[CBSE SQP Marking Scheme, 2020] 1

Detailed Solution:

$\sin\alpha = \frac{\sqrt{3}}{2}$

$\sin\alpha = \sin 60° \quad [\because \sin 60° = \frac{\sqrt{3}}{2}]$

$$\alpha = 60°$$

and $\cos\beta = 0$

$$\cos\beta = \cos 90° \quad [\because \cos 90° = 0] \ ½$$

$$\beta = 90°$$

Now, $\beta - \alpha = 90° - 60°$

$$= 30°. \quad ½$$

Q. 9. Find A, if $\tan 2A = \cot (A - 24°)$.

A [CBSE Delhi Set-I, II, III, 2019]
[CBSE Delhi/OD 2018]
[Board Term-I, 2016]

Sol. $\tan 2A = \cot(90° - 2A)$ ½

$90° - 2A = A - 24°$ ½

$\Rightarrow A = 38°$

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given, $\tan 2A = \cot (A - 24°)$

$\Rightarrow \cot (90° - 2A) = \cot (A - 24°)$ ½

$[\therefore \tan\theta = \cot (90° - \theta)]$

On comparing angles, we get

$$90° - 2A = A - 24°$$

$\therefore 3A = 90° + 24°$

$\therefore 3A = 114°$

$\therefore A = \frac{114°}{3} = 38°$ ½

Hence, angle $A = 38°$.

AI **Q. 10. Evaluate: $\sin^2 60° + 2\tan 45° - \cos^2 30°$**

A [CBSE OD Set-I, II, 2019]

Sol. $\sin^2 60° + 2\tan 45° - \cos^2 30°$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2$$

[For any two correct values] ½

$= 2$ [CBSE Marking Scheme, 2019] ½

Q. 11. What is the value of $(\cos^2 67° - \sin^2 23°)$?

U [CBSE Delhi/OD, 2018]

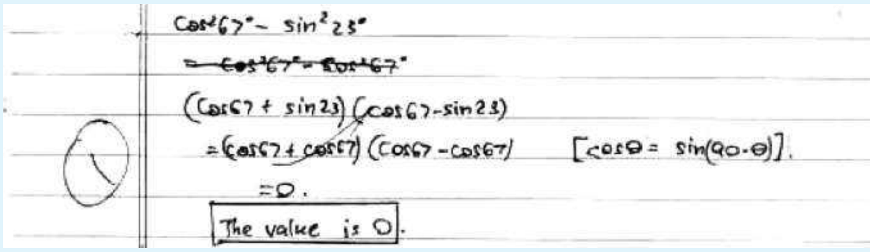
Sol. $\because \cos^2 67° = \cos^2 (90° - 23°) = \sin^2 23°$

$\therefore \sin^2 23° - \sin^2 23° = 0$

[CBSE Marking Scheme, 2018] 1

Detailed Solution:

Topper Answer, 2018



$\cos^2 67° - \sin^2 23°$

~~$= \cos^2 67° - \cos^2 67°$~~

$(\cos 67 + \sin 23)(\cos 67 - \sin 23)$

$= (\cos 67 + \cos 67)(\cos 67 - \cos 67)$ $[\cos\theta = \sin(90 - \theta)]$

$= 0.$

The value is 0.

1

Q. 12. If $\tan 2A = \cot (A - 18°)$, where $2A$ is an acute angle, find the value of A. C + U [CBSE Delhi/OD, 2018]

Sol. $\tan 2A = \cot (A - 18°)$

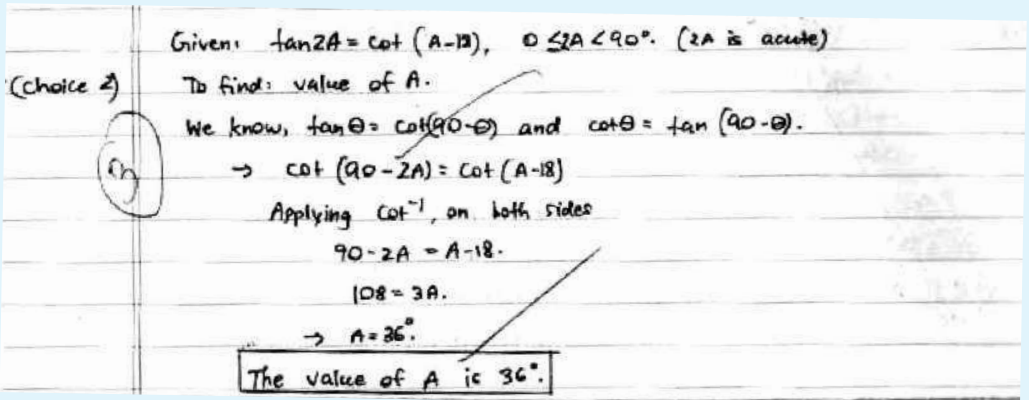
$\Rightarrow 90° - 2A = A - 18°$ ½

$\Rightarrow 3A = 108°$ ½

$\Rightarrow A = 36°$ [CBSE Marking Scheme, 2018]

Detailed Solution:

Topper Answer, 2018



(Choice 2)

Given: $\tan 2A = \cot (A - 18)$, $0 \le 2A < 90°$. (2A is acute)

To find: value of A.

We know, $\tan\theta = \cot(90 - \theta)$ and $\cot\theta = \tan(90 - \theta)$.

$\rightarrow \cot (90 - 2A) = \cot (A - 18)$

Applying $\cot^{-1}$, on both sides

$90 - 2A = A - 18.$

$108 = 3A.$

$\rightarrow A = 36°.$

The value of A is 36°.

1

COMMONLY MADE ERROR

- Generally conversion from tan to cot is not done and the angles are equated and simplified incorrectly.

ANSWERING TIP

- The candidates should remember to convert the tan to cot before equating the angles.

Q. 13. If sin θ = cos θ, then find the value of 2 tan θ + cos² θ. U [CBSE SQP, 2018]

Sol. Given, $\sin\theta = \cos\theta \quad \theta = 45°$

$$2\tan\theta + \cos^2\theta = 2 + \frac{1}{2} = \frac{5}{2}$$

[CBSE Marking Scheme, 2018] **1**

Detailed Solution:

We have, $\sin\theta = \cos\theta$

We know that, $\cos\theta = \sin(90° - \theta)$

$\therefore \quad \sin\theta = \sin(90° - \theta)$

$\Rightarrow \quad \theta = 90° - \theta$

$\Rightarrow \quad 2\theta = 90°$

$\Rightarrow \quad \theta = 45°.$ ½

Now, $2\tan\theta + \cos^2\theta = 2\tan 45° + \cos^2 45°$

$$= 2 \times 1 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 2 + \frac{1}{2} = \frac{5}{2}.$$ ½

Q. 14. If sec θ.sin θ = 0, then find the value of θ. R [Board Term-1, 2016]

Sol. Given, $\sec\theta.\sin\theta = 0$

or, $\dfrac{\sin\theta}{\cos\theta} = 0$

or, $\tan\theta = 0 = \tan 0°$

$\therefore \quad \theta = 0°$ **1**

[CBSE Marking Scheme, 2016]

Q. 15. Find the value of $\dfrac{\sin 25°}{\cos 65°} + \dfrac{\tan 23°}{\cot 67°}$. U [Board Term-1, 2015]

Sol. $$\frac{\sin 25°}{\cos 65°} + \frac{\tan 23°}{\cot 67°} = \frac{\sin 25°}{\cos(90° - 25°)} + \frac{\tan 23°}{\cot(90° - 23°)}$$

$= 1 + 1 = 2$ **1**

[CBSE Marking Scheme, 2015]

Q. 16. If cos 2A = sin (A – 15°), find A. U [Board Term-1, 2015]

Sol. $\sin(90° - 2A) = \sin(A - 15°)$

or, $90° - 2A = A - 15°$

or, $3A = 105°$

$\therefore \quad A = 35°$ **1**

[CBSE Marking Scheme, 2015]

Q. 17. If tan (3x + 30°) = 1, then find the value of x. U [Board Term-1, 2015]

Sol. $\tan(3x + 30°) = 1 = \tan 45°$

or, $3x + 30° = 45°$ or, $x = 5°$ **1**

[CBSE Marking Scheme, 2015]

Q. 18. What happens to value of cos θ when θ increases from 0° to 90° ? A [Board Term-1, 2015]

Sol. cos θ decreases from 1 to 0. **1**

[CBSE Marking Scheme, 2015]

Short Answer Type Questions-I — 2 marks each

AI **Q. 1. If $\tan A = \dfrac{3}{4}$, find the value of $\dfrac{1}{\sin A} + \dfrac{1}{\cos A}$.** R [CBSE SQP, 2020-21]

Sol. Given that, $\tan A = \dfrac{3}{4} = \dfrac{3k}{4k}$ ½

$$\sin A = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos A = \frac{4k}{5k} = \frac{4}{5}$$ ½

$$\frac{1}{\sin A} + \frac{1}{\cos A} = \frac{5}{3} + \frac{5}{4}$$ ½

$$= \frac{20 + 15}{12}$$

$$= \frac{35}{12}$$ ½

[CBSE Marking Scheme, 2020-21]

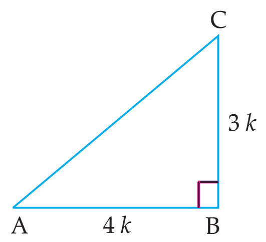
Detailed Solution:

We have, $\tan A = \dfrac{3}{4}$

$$= \frac{\text{Perpendicular}}{\text{Base}}$$

i.e., perpendicular = $3k$ and base = $4k$.

Let ABC be a right angled triangle, then $BC = 3k$ and $AB = 4k$



Now $AC^2 = AB^2 + BC^2$

(By using Pythagoras theorem)

$$= (4k^2) + (3k)^2$$
$$= 16k^2 + 9k^2$$
$$= 25k^2$$
$\Rightarrow$ $AC = 5k$ ½

Now, $\sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$ ½

and $\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$ ½

Hence, $\frac{1}{\sin A} + \frac{1}{\cos A} = \frac{1}{3/5} + \frac{1}{4/5} = \frac{5}{3} + \frac{5}{4}$

$$= \frac{20+15}{12} = \frac{35}{12}.$$ ½

AI **Q. 2. If $\sqrt{3}$ sin θ – cos θ = 0 and 0° < θ < 90°, find the value of θ.** U [Board SQP, 2020-21]

Sol. Here $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0° < \theta < 90°$

or, $\sqrt{3}\sin\theta = \cos\theta$

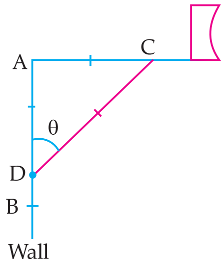
or, $\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$ 1

or, $\tan\theta = \frac{1}{\sqrt{3}}$

$= \tan 30°$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$\therefore$ $\theta = 30°.$ 1

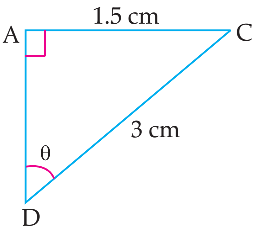
AI **Q. 3. The rod *AC* of TV disc antenna is fixed at right angles to wall *AB* and a rod *CD* is supporting the disc as shown in figure. If *AC*=1.5 m long and *CD* = 3 m, find (i) tan θ and (ii) sec θ + cosec θ.**



C + R [CBSE OD Set-II, 2020]

Sol. Given, $AC = 1.5$ m

and $CD = 3$ m



½

In right angled triangle *CAD*,

$AD^2 + AC^2 = DC^2$ (Using Pythagoras theorem)

$\Rightarrow AD^2 + (1.5)^2 = (3)^2$

$\Rightarrow$ $AD^2 = 9 - 2.25 = 6.75$

$\Rightarrow$ $AD = \sqrt{6.75} = 2.6$ m (Approx) ½

(i) $\tan\theta = \frac{AC}{AD} = \frac{1.5}{2.6} = \frac{15}{26}$ ½

(ii) $\sec\theta + \text{cosec}\,\theta = \frac{CD}{AD} + \frac{CD}{AC}$

$$= \frac{3}{2.6} + \frac{3}{1.5} = \frac{41}{13}.$$ ½

Q. 4. *A*, *B*, *C* are interior angles of ΔABC. Prove that

$$\text{cosec}\left(\frac{A+B}{2}\right) = \sec\frac{C}{2}$$

U [CBSE Comptt. Set-I, II, III, 2018]

Sol. $A + B + C = 180°$

$\frac{A+B}{2} = 90° - \frac{C}{2}$ 1

$\text{cosec}\left(\frac{A+B}{2}\right) = \text{cosec}\left(90° - \frac{C}{2}\right) = \sec\frac{C}{2}$ 1

[CBSE Marking Scheme, 2018]

Detailed Solution:

As we know that the sum of interior angles of a triangle is 180°.

$\therefore$ $A + B + C = 180°$ ½

$\Rightarrow$ $A + B = 180° - C$

Dividing by 2 on both sides,

$$\frac{A+B}{2} = \frac{180-C}{2}$$

$$= 90° - \frac{C}{2}$$ ½

Multiplying by cosec on both sides,

$$\text{cosec}\left(\frac{A+B}{2}\right) = \text{cosec}\left(90° - \frac{C}{2}\right)$$

$\Rightarrow$ $\text{cosec}\left(\frac{A+B}{2}\right) = \sec\frac{C}{2}.$ **Hence Proved 1**

AI **Q. 5. Evaluate:**

$$\frac{3\tan^2 30° + \tan^2 60° + \text{cosec}\,30° - \tan 45°}{\cot^2 45°}$$

U [Board Term-1, 2016]

Sol. $\frac{3\tan^2 30° + \tan^2 60° + \text{cosec}\,30° - \tan 45°}{\cot^2 45°}$

$$= \frac{3\times\left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 2 - 1}{(1)^2}$$ 1

$$= \frac{3\times\frac{1}{3} + 3 + 2 - 1}{1}$$

$= 1 + 3 + 2 - 1 = 5$ 1

[CBSE Marking Scheme, 2016]

COMMONLY MADE ERROR

- Sometimes students get confused with the values of trigonometric angles. They substitute wrong values which leads to the wrong result.

ANSWERING TIP

- Memorize the values of trigonometric angles properly and practice more such problems to not to get confused.

Q. 6. If sin ($A + B$) = 1 and sin ($A - B$) = $\frac{1}{2}$, $0 \le A + B \le 90°$ and $A > B$, then find A and B.

U [Board Term-1, 2016]

Sol. Here, $\sin(A + B) = 1 = \sin 90°$

or, $A + B = 90°$ ...(i)

$\sin(A - B) = \frac{1}{2} = \sin 30°$ **1**

or, $A - B = 30°$ ...(ii)

Solving eq. (i) and (ii),

$A = 60°$ and $B = 30°$ **1**

[CBSE Marking Scheme, 2016]

AI **Q. 7. Find the value of :**

sin 30°. cos 60° + cos 30°. sin 60°

Is it equal to sin 90° or cos 90° ?

U [Board Term-1, 2016]

Sol. sin 30° cos 60° + cos 30° sin 60°

$$= \frac{1}{2}\times\frac{1}{2}+\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}$$ **1**

$$= \frac{1}{4}+\frac{3}{4}$$

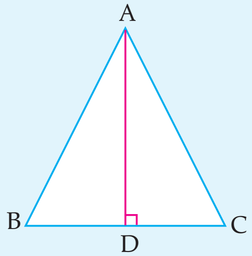
$$= \frac{4}{4} = 1$$ **1**

It is equal to sin 90° = 1 but not equal to cos 90° as cos 90° = 0. [CBSE Marking Scheme, 2016]

Q. 8. Find cosec 30° and cos 60° geometrically.

U [Board Term-1, 2015]

Sol.



Let a triangle ABC with each side equal to $2a$. ½

$\angle A = \angle B = \angle C = 60°$

Draw AD perpendicular to BC

$\Delta BDA \cong \Delta CDA$ (by RHS) ½

$BD = CD$

$\angle BAD = \angle CAD = 30°$ (by c.p.c.t)

In ΔBDA, $\text{cosec } 30° = \frac{AB}{BD} = \frac{2a}{a} = 2$ ½

and $\cos 60° = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$ ½

[CBSE Marking Scheme, 2015]

Short Answer Type Questions-II — 3 marks each

AI **Q. 1. Evaluate:**

$$\frac{\cos^2(45^o+\theta)+\cos^2(45^o-\theta)}{\tan(60^o+\theta)\times\tan(30^o-\theta)}$$

+ (cos 30° + sin 90°) × (tan 60° – sec 0°)

A [CBSE SQP 2020]

Sol. $$\frac{\cos^2(45^o+\theta)+\cos^2(45^o-\theta)}{\tan(60^o+\theta)\times\tan(30^o-\theta)}$$

+ (cot 30° + sin 90°) × (tan 60° – sec 0°)

$$= \frac{\cos^2(45^o+\theta)+\sin^2(45^o+\theta)}{\tan(60^o+\theta)\times\cot(60^o+\theta)} + (\sqrt{3}+1)\times(\sqrt{3}-1)$$ **2**

= 1 + 2 = 3 [CBSE SQP Marking Scheme, 2020] **1**

Detailed Solution:

Given, $$\frac{\cos^2(45^o+\theta)+\cos^2(45^o-\theta)}{\tan(60^o+\theta)\times\tan(30^o-\theta)}$$

+ (cot 30° + sin 90°) × (tan 60° – sec 0°)

$$= \frac{\cos^2(45^o+\theta)+\{\cos[90^o-(45^o+\theta)\}^2}{\tan(60^o+\theta)\times\tan[90^o-(60^o+\theta)]}$$

+ (cot 30° + sin 90°) × (tan 60° – sec 0°) **1**

$$= \frac{\cos^2(45^o+\theta)+\sin^2(45^o+\theta)}{\tan(60^o+\theta)\times\cot(60^o+\theta)}$$

+ (cot 30° + sin 90°) × (tan 60° – sec 0°) ½

$$= \frac{1}{\tan(60°+\theta)\times\frac{1}{\tan(60°+\theta)}}$$

+ (cot 30° + sin 90°) × (tan 60° – sec 0°)

[$\because \sin^2\theta + \cos^2\theta = 1$] ½

$$= 1+(\sqrt{3}+1)\times(\sqrt{3}-1)$$

[$\because (a+b)(a-b) = a^2 - b^2$] ½

= 1 + 3 – 1

= 3. ½

Q. 2. If $\tan(A + B) = 1$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, $0° < A + B < 90°$, $A > B$, then find the values of A and B.

[R] [CBSE Delhi Region, 2019]

Topper Answer, 2017

Sol. Given, $\tan(A+B) = 1$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\tan(A+B) = 1.$

$\Rightarrow \tan(A+B) = \tan 45°$

$\Rightarrow A+B = 45°$ —— (1)

Now taking,

$\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B) = \tan 30°$

$\Rightarrow A - B = 30°$ —— (2)

Adding (1) and (2);

$A+B+A-B = 45° + 30°$

$\Rightarrow 2A = 75° \Rightarrow A = \frac{75°}{2} \Rightarrow A = 37.5°$

$B = 45° - A \Rightarrow B = 45° - 37.5° \Rightarrow B = 7.5°$

$\therefore$ $A = 37.5°$, $B = 7.5°$ 3

[AI] Q. 3. Evaluate:

$$\left(\frac{3\sin 43^o}{\cos 47^o}\right)^2 - \frac{\cos 37^o \text{ cosec } 53^o}{\tan 5^o \tan 25^o \tan 45^o \tan 65^o \tan 85^o}$$

[A] [CBSE OD Set-I, III, 2019]

Sol. $$\left(\frac{3\sin 43^o}{\cos 47^o}\right) - \frac{\cos 37^o \text{ cosec } 53^o}{\tan 5^o \tan 25^o \tan 45^o \tan 65^o \tan 85^o}$$

$$= \left(\frac{3\sin 43^o}{\cos(90^o - 43^o)}\right)^2$$

$$- \frac{\cos 37^o \text{ cosec } (90^o - 37^o)}{\tan 5^o \tan 25^o (1) \tan(90^o - 25^o) \tan(90^o - 5^o)} \quad 1$$

$$= \left(\frac{3\sin 43^o}{\sin 43^o}\right)^2 - \frac{\cos 37^o \sec 37^o}{\tan 5^o \tan 25^o (1) \cot 25^o \cot 5^o} \quad 1$$

$$= 9 - \frac{1}{1} = 8$$ [CBSE Marking Scheme, 2019] 1

Detailed Solution:

LHS =

$$\left(\frac{3\sin 43^o}{\cos 47^o}\right)^2 - \frac{\cos 37^o \text{ cosec } 53^o}{\tan 5^o \tan 25^o \tan 45^o \tan 65^o \tan 85^o}$$

$$= \left(\frac{3\sin 43^o}{\cos(90^o - 43^o)}\right)^2$$

$$- \frac{\cos 37^o \text{ cosec } (90^o - 37^o)}{\tan 5^o \tan 25^o \times 1 \times \tan(90^o - 25^o) \tan(90^o - 5^o)} \quad 1$$

$$= \left(\frac{3\sin 43^o}{\sin 43^o}\right)^2 - \frac{\cos 37^o \times \sec 37^o}{\tan 5^o \tan 25^o \cot 25^o \cot 5^o} \quad 1$$

$$= 9 - \frac{\cos 37^o \times \frac{1}{\cos 37^o}}{\tan 5^o \tan 25^o \times \frac{1}{\tan 25^o} \times \frac{1}{\tan 5^o}} \quad ½$$

$$= 9 - \frac{1}{1} = 9 - 1$$

$$= 8. \quad ½$$

Q. 4. If 4 tan θ = 3, evaluate $\left(\dfrac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}\right)$

U [CBSE Delhi/OD, 2018]

Sol. Given, $4\tan\theta = 3$

$\Rightarrow \quad \tan\theta = \dfrac{3}{4}$

$\Rightarrow \quad \sin\theta = \dfrac{3}{5}$ and $\cos\theta = \dfrac{4}{5}$ ½

$\therefore \left(\dfrac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}\right) = \dfrac{4\times\frac{3}{5}-\frac{4}{5}+1}{4\times\frac{3}{5}+\frac{4}{5}-1}$ **1**

$= \dfrac{13}{11}$ **1**

[CBSE Marking Scheme, 2018]

Detailed Solution:

We have $4\tan\theta = 3$

$\Rightarrow \quad \tan\theta = \dfrac{3}{4}$

We have, $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}$

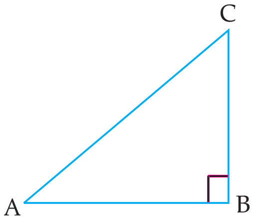
Let perpendicular $= 3x$ and

Base $= 4x$

Also let ABC be a right angled Δ.

$AC = \sqrt{AB^2+BC^2}$

(By using Pythagoras theorem)



$\therefore \quad AC = \sqrt{16x^2+9x^2}$

$= \sqrt{25x^2} = 5x$

Then $\sin\theta = \dfrac{BC}{AC} = \dfrac{3x}{5x} = \dfrac{3}{5}$

and $\cos\theta = \dfrac{AB}{AC} = \dfrac{4x}{5x} = \dfrac{4}{5}$ **1**

$\therefore \quad \dfrac{4\sin\theta-\cos\theta+1}{4\cos\theta+\cos\theta-1} = \dfrac{4\times\frac{3}{5}-\frac{4}{5}+1}{4\times\frac{3}{5}+\frac{4}{5}-1}$

$= \dfrac{\frac{8}{5}+1}{\frac{16}{5}-1} = \dfrac{\frac{13}{5}}{\frac{11}{5}}$

$= \dfrac{13}{11}.$ **1**

Alternative Method:

Given $4\tan\theta = 3$

$\tan\theta = \dfrac{3}{4}$

$\tan^2\theta = \dfrac{9}{16}$

$\sec^2\theta = 1+\tan^2\theta = 1+\dfrac{9}{16}$

$\sec\theta = \sqrt{\dfrac{25}{16}} = \dfrac{5}{4}$

$\dfrac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}$

Divide by cos θ

$= \dfrac{4\tan\theta-1+\sec\theta}{4\tan\theta+1-\sec\theta}$

$= \dfrac{3-1+\frac{5}{4}}{3+1-\frac{5}{4}}$

$= \dfrac{2+\frac{5}{4}}{4-\frac{5}{4}} = \dfrac{8+5}{16-5} = \dfrac{13}{11}$

COMMONLY MADE ERROR

- Mostly candidates do not find the values of sine and cosine. Some candidates do the wrong calculation.

ANSWERING TIP

- Candidates should find the value of sin θ and cos θ by using Pythagoras theorem.

Q. 5. If sin (*A* + 2*B*) = $\dfrac{\sqrt{3}}{2}$ **and cos(*A* + 4*B*) = 0, *A* > *B*, and *A* + 4*B* ≤ 90°, then find *A* and *B*.**

C + U [CBSE Comptt. Set-I, II, III, 2018]

Sol. Given, $\sin(A+2B) = \dfrac{\sqrt{3}}{2} \Rightarrow A+2B = 60°$ **1**

$\Rightarrow \quad \cos(A+4B) = 0, \Rightarrow A+4B = 90°$ **1**

Solving, we get $A = 30°$ and $B = 15°$ ½ + ½

[CBSE Marking Scheme, 2018]

Detailed Solution:

We have $\sin(A+2B) = \dfrac{\sqrt{3}}{2}$

$\therefore \quad \sin(A+2B) = \sin 60°$ ½

$\Rightarrow \quad A+2B = 60°$ ...(i) ½

and $\cos(A + 4B) = 0$

$\therefore$ $\cos(A + 4B) = \cos 90°$ $[\because \cos 90° = 0]$ ½

$\Rightarrow$ $A + 4B = 90°$ ...(ii) ½

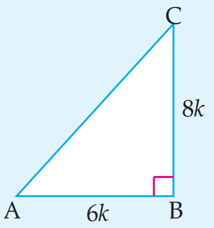
Solving eq. (i) and (ii), we get

$A = 30°$ and $B = 15°$. **1**

Q. 6. If in a triangle ABC right angled at B, $AB = 6$ units and $BC = 8$ units, then find the value of $\sin A.\cos C + \cos A.\sin C$. U [Board Term-1, 2016]

Sol. Here, $AC^2 = (8k)^2 + (6k)^2 = 100k^2$

or, $AC = 10k$



$\therefore$ $\sin A = \frac{8k}{10k}$, $\cos A = \frac{6}{10}$ **1**

and $\sin C = \frac{6k}{10k}$, $\cos C = \frac{8}{10}$ **1**

$\therefore$ $\sin A \cos C + \cos A \sin C$

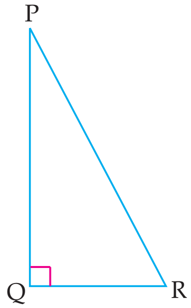
$$= \frac{8}{10}\times\frac{8}{10}+\frac{6}{10}\times\frac{6}{10}$$

$$= \frac{64}{100}+\frac{36}{100}$$

$$= \frac{100}{100} = 1.$$ **1**

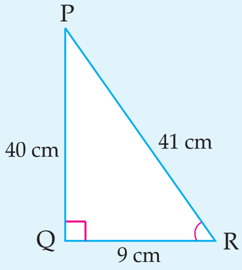
[CBSE Marking Scheme, 2016]

Q. 7.



In the given ΔPQR, right–angled at Q, $QR = 9$ cm and $PR - PQ = 1$ cm. Determine the value of $\sin R + \cos R$. U [Board Term-1, 2015]

Sol.



$PQ^2 + QR^2 = PR^2$

(By Pythagoras theorem)

or, $PQ^2 + 9^2 = PR^2$

or, $PQ^2 + 81 = (PQ + 1)^2$

or, $PQ^2 + 81 = PQ^2 + 1 + 2PQ$

or, $PQ = 40$

$PR - PQ = 1$ (Given)

or, $PR = 1 + 40$

or, $PR = 41$

$\therefore$ $\sin R + \cos R = \frac{40}{41} + \frac{9}{41} = \frac{49}{41}$ **3**

[CBSE Marking Scheme, 2015]

Long Answer Type Questions — 5 marks each

Q. 1. Evaluate:

$\tan^2 30° \sin 30° + \cos 60° \sin^2 90° \tan^2 60° - 2 \tan 45° \cos^2 0° \sin 90°$ R [Board Term-1, 2015]

Sol. $\tan^2 30° \sin 30° + \cos 60° \sin^2 90° \tan^2 60 - 2 \tan 45° \cos^2 0° \sin 90°$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 \times\frac{1}{2}+\frac{1}{2}\times(1)^2\times\left(\sqrt{3}\right)^2-2\times1\times1^2\times1$$

$$= \frac{1}{3}\times\frac{1}{2}+\frac{1}{2}\times 1\times 3-2\times 1\times 1\times 1.$$

$$= \frac{1}{6}+\frac{3}{2}-2 = \frac{1+9-12}{6}$$

$$= -\frac{2}{6}=-\frac{1}{3}$$ [CBSE Marking Scheme, 2015] **5**

Q. 2. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\cot^2\theta + \tan^2\theta$. U

Sol. Let $\cot\theta = x$,

then $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$ becomes

$\sqrt{3}x^2 - 4x + \sqrt{3} = 0$ **1**

or, $\sqrt{3}x^2 - 3x - x + \sqrt{3} = 0$

or, $(x - \sqrt{3})(\sqrt{3}x - 1) = 0$

$\therefore$ $x = \sqrt{3}$ or $\frac{1}{\sqrt{3}}$ **1**

or, $\cot\theta = \sqrt{3}$ or $\cot\theta = \frac{1}{\sqrt{3}}$ **1**

$\therefore$ $\theta = 30°$ or $\theta = 60°$

If $\theta = 30°$, then

$$\cot^2 30° + \tan^2 30° = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \frac{1}{3} = \frac{10}{3}$$ **1**

If θ = 60°, then

$$\cot^2 60° + \tan^2 60° = \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$$

$$= \frac{1}{3} + 3 = \frac{10}{3} \qquad 1$$

AI **Q. 3. In an acute angled triangle ABC, if $\sin(A + B - C) = \frac{1}{2}$ and $\cos(B + C - A) = \frac{1}{\sqrt{2}}$, find $\angle A$, $\angle B$ and $\angle C$.** A

Sol. We have

$$\sin(A + B - C) = \frac{1}{2} = \sin 30°$$

or, $A + B - C = 30°$ ...(i) **1**

and $\cos(B + C - A) = \frac{1}{\sqrt{2}} = \cos 45°$

or, $B + C - A = 45°$ ...(ii) **1**

Adding eqns. (i) and (ii), we get

$$2B = 75°$$

or, $B = 37.5°$ **1**

Now subtracting eqn. (ii) from eqn. (i),

$$2(A - C) = -15°$$

or, $A - C = -7.5°$ ...(iii)

$\because$ $A + B + C = 180°$ **1**

or, $A + C = 142.5°$ ...(iv)

Adding eqns. (iii) and (iv),

$$2A = 135°$$

or, $A = 67.5°$

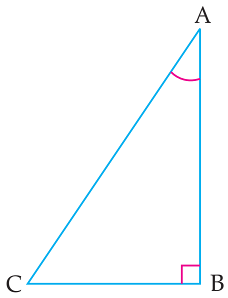
and $C = 75°$

Hence, $\angle A = 67.5°, \angle B = 37.5°$ and $\angle C = 75°$ **1**

TOPIC - 2
Trigonometric Identities

Revision Notes

- An equation is called an identity if it is true for all values of the variable(s) involved.
- An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all values of the angle.



In Δ ABC, right-angled at B, By Pythagoras Theorem,

$$AB^2 + BC^2 = AC^2 \qquad ...(i)$$

Dividing each term of (i) by AC^2,

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

or $$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = \left(\frac{AC}{AC}\right)^2$$

or $$(\cos A)^2 + (\sin A)^2 = 1$$

or $$\cos^2 A + \sin^2 A = 1 \qquad ...(ii)$$

This is true for all values of A such that $0° \le A \le 90°$. So, this is a trigonometric identity. Now divide eqn.(i) by AB^2.

$$\frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

or $$\left(\frac{AB}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2 = \left(\frac{AC}{AB}\right)^2$$

or $$1 + \tan^2 A = \sec^2 A \qquad ...(iii)$$

Is this equation true for $A = 0°$? Yes, it is. What about $A = 90°$? Well, $\tan A$ and $\sec A$ are not defined for $A = 90°$. So, eqn. (iii) is true for all values of A such that $0° \le A < 90°$.

Again dividing eqn. (i) by BC^2.

$$\frac{AB^2}{BC^2} + \frac{BC^2}{BC^2} = \frac{AC^2}{BC^2}$$

or $$\left(\frac{AB}{BC}\right)^2 + \left(\frac{BC}{BC}\right)^2 = \left(\frac{AC}{BC}\right)^2$$

or $$\cot^2 A + 1 = \text{cosec}^2 A \quad \text{...(iv)}$$

Note that cosec A and cot A are not defined for all $A = 0°$. Therefore eqn. (iv) is true for all value of A such that $0° < A \leq 90°$.

Using these identities, we can express each trigonometric ratio in terms of other trigonometric ratios, *i.e.*, if any one of the ratios is known, we can determine the values of other trigonometric ratios.

How is it done on the GREENBOARD?

Q.1. Prove that. $\dfrac{1}{\sec\theta - \tan\theta} = \dfrac{1+\sin\theta}{\cos\theta}$

Solution

Step I: L.H.S. $= \dfrac{1}{\sec\theta - \tan\theta}$

Multiplying with $\sec\theta + \tan\theta$

$$\text{L.H.S.} = \frac{1}{\sec\theta - \tan\theta} \times \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$$

$$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$[\because (a+b)(a-b) = a^2 - b^2]$

$$= \sec\theta + \tan\theta$$

$[\because \sec^2\theta - \tan^2\theta = 1]$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta} = \text{R.H.S.}$$

Very Short Answer Type Questions — 1 mark each

AI **Q. 1. If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$, then find the value of $x + y$.** C [CBSE SQP, 2020-21]

Sol. $x + y = 2\sin^2\theta + 2\cos^2\theta + 1$ ½

$= 2(\sin^2\theta + \cos^2\theta) + 1$

(As $\sin^2 x + \cos^2 x = 1$)

$= 3.$ ½

[CBSE Marking Scheme, 2020-21]

Detailed Solution:

We have $x = 2\sin^2\theta$

and $y = 2\cos^2\theta + 1$

Then, $x + y = 2\sin^2\theta + 2\cos^2\theta + 1$ ½

$= 2(\sin^2\theta + \cos^2\theta) + 1$

$= 2 \times 1 + 1$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= 2 + 1 = 3.$ ½

AI **Q. 2. Find the value of $\left(\sin^2\theta + \dfrac{1}{1+\tan^2\theta}\right)$** U [CBSE Delhi Set-I, 2020]

Sol. $\sin^2\theta + \dfrac{1}{1+\tan^2\theta} = \sin^2\theta + \dfrac{1}{\sec^2\theta}$

$[\because 1 + \tan^2\theta = \sec^2\theta]$ ½

$= \sin^2\theta + \cos^2\theta$

$\left[\because \dfrac{1}{\sec\theta} = \cos\theta\right]$

$= 1.$ $[\because \sin^2\theta + \cos^2\theta = 1]$ ½

AI **Q. 3. Find the value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$.** U [CBSE Delhi Set-I, 2020]

Sol. $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$[\because 1 + \tan^2\theta = \sec^2\theta]$

$= \sec^2\theta(1 - \sin\theta)(1 + \sin\theta)$

$= \sec^2\theta(1 - \sin^2\theta)$

$[\because (a-b)(a+b) = a^2 - b^2]$ ½

$= \sec^2\theta \times \cos^2\theta$ $[\because 1 - \sin^2\theta = \cos^2\theta]$

$= \dfrac{1}{\cos^2\theta} \times \cos^2\theta$ $\left[\because \sec\theta = \dfrac{1}{\cos\theta}\right]$

$= 1.$ ½

[CBSE Marking Scheme, 2020]

AI **Q. 4. If $\sin A + \sin^2 A = 1$, then find the value of the expression $(\cos^2 A + \cos^4 A)$.** C + U [CBSE OD Set-I, 2020]

Sol. Given, $\sin A + \sin^2 A = 1$

$\Rightarrow \sin A = 1 - \sin^2 A = \cos^2 A$ ½

On squaring both sides, we get

$$\sin^2 A = \cos^4 A$$

$$\Rightarrow \quad 1 - \cos^2 A = \cos^4 A$$

$$\Rightarrow \quad \cos^2 A + \cos^4 A = 1. \qquad ½$$

Q. 5. Find the value of sin 23° cos 67° + cos 23° sin 67°.

R [CBSE OD Set-II, 2020]

Sol. $\sin 23° \cos 67° + \cos 23° \sin 67°$

$= \sin 23° \cos (90° - 23°) + \cos 23° \sin (90° - 23°)$

$= \sin 23° \sin 23° + \cos 23° \cos 23°$

$[\because \cos (90° - \theta) = \sin \theta \text{ and } \sin (90° - \theta) = \cos \theta]$ ½

$= \sin^2 23° + \cos^2 23°$

$= 1.$ $[\sin^2 A + \cos^2 A = 1]$ ½

Q. 6. Find the value of sin 32° cos 58° + cos 32° sin 58°.

R [CBSE OD Set-III, 2020]

Sol. $\sin 32° \cos 58° + \cos 32° \sin 58°$

$= \sin 32° \cos (90° - 32°) + \cos 32° \sin (90° - 32°)$

$= \sin 32° \sin 32° + \cos 32° \cos 32°$

$[\because \cos (90° - \theta) = \sin \theta \text{ and } \sin (90° - \theta) = \cos \theta]$ ½

$= \sin^2 32° + \cos^2 32°$

$= 1.$ $[\sin^2 \theta + \cos^2 \theta = 1]$ ½

Q. 7. If $\sin \theta + \cos \theta = \sqrt{2} \cos \theta$, $(\theta \neq 90°)$ then the value of tan θ is :

A [CBSE SQP, 2020]

Sol. $\sqrt{2} - 1$ [CBSE SQP Marking Scheme, 2020]

Detailed Solution:

$$\sin \theta + \cos \theta = \sqrt{2} \cos \theta$$

or, $$\frac{\sin\theta + \cos\theta}{\cos\theta} = \sqrt{2} \qquad ½$$

or, $$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} = \sqrt{2}$$

or, $$\tan \theta + 1 = \sqrt{2}$$

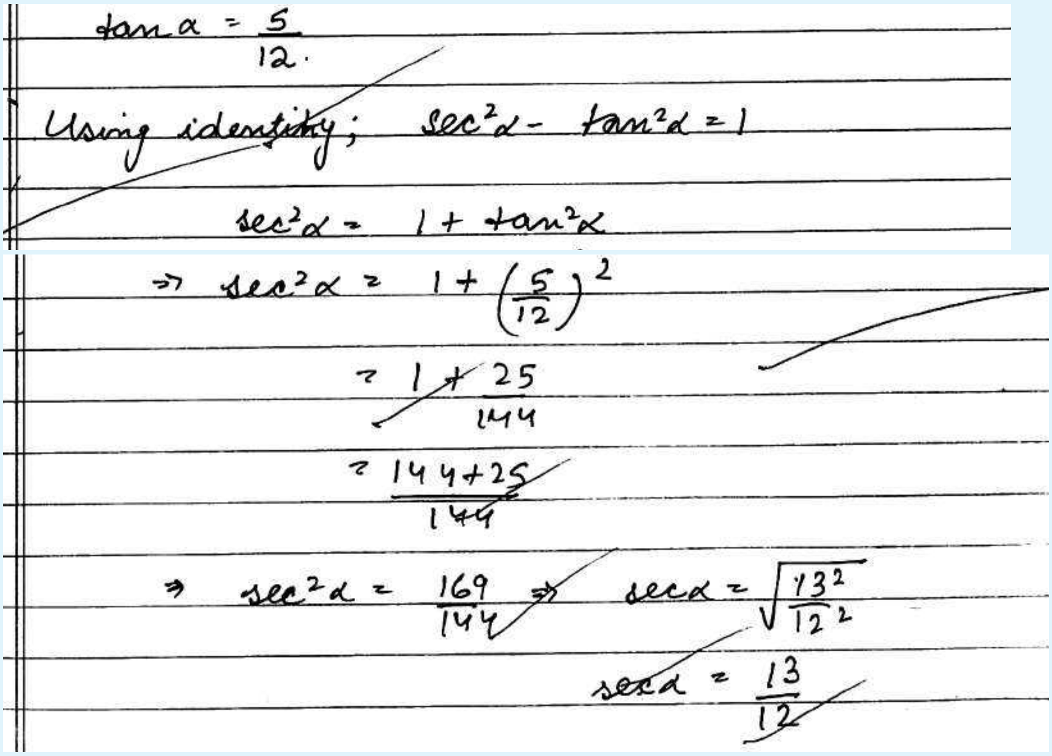
or, $$\tan \theta = \sqrt{2} - 1. \qquad ½$$

Q. 8. If $\tan \alpha = \frac{5}{12}$, find the value of sec α.

[CBSE Delhi Region, 2019]

Topper Answer, 2019

Sol.



tan α = 5/12.

Using identity; sec²α − tan²α = 1

sec²α = 1 + tan²α

⇒ sec²α = 1 + (5/12)²

= 1 + 25/144

= (144+25)/144

⇒ sec²α = 169/144 ⇒ sec α = √(13²/12²)

sec α = 13/12

1

Q. 9. Write the value of $\cot^2 \theta - \frac{1}{\sin^2 \theta}$

U [CBSE SQP, 2018]

Sol. $$\cot^2 \theta - \frac{1}{\sin^2 \theta} = \cot^2 \theta - \operatorname{cosec}^2 \theta$$

$$= \cot^2 \theta - 1 - \cot^2 \theta$$

$$= -1 \qquad 1$$

[CBSE Marking Scheme, 2018]

Q. 10. If $k + 1 = \sec^2 \theta(1 + \sin \theta)(1 - \sin \theta)$, then find the value of k. C + U [Board Term-1, 2015]

Sol. $k + 1 = \sec^2 \theta(1 + \sin \theta)(1 - \sin \theta)$

or, $k + 1 = \sec^2 \theta(1 - \sin^2 \theta)$

or, $k + 1 = \sec^2 \theta . \cos^2 \theta$ $[\because \sin^2 \theta + \cos^2 \theta = 1]$

or, $k + 1 = \sec^2 \theta \times \frac{1}{\sec^2 \theta}$

or, $k + 1 = 1$ ½

or, $k = 1 - 1$ ½

$\therefore$ $k = 0.$ [CBSE Marking Scheme, 2015]

Short Answer Type Questions-I — 2 marks each

AI **Q. 1. Prove that:** $1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha} = \operatorname{cosec}\alpha$.

A [CBSE OD Set-I, 2020]

Sol.
$$\text{L.H.S} = 1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha}$$
$$= 1 + \frac{\operatorname{cosec}^2\alpha - 1}{1+\operatorname{cosec}\alpha}$$
$$= 1 + \frac{(1+\operatorname{cosec}\alpha)(\operatorname{cosec}\alpha - 1)}{1+\operatorname{cosec}\alpha} \quad \mathbf{1}$$
$$= 1 + \operatorname{cosec}\alpha - 1$$
$$= \operatorname{cosec}\alpha = \text{R.H.S.}$$
$\therefore$ L.H.S = R.H.S. **Hence Proved. 1**

AI **Q. 2. Show that** $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

A [CBSE OD Set-I, 2020]

Sol.
$$\text{L.H.S.} = \tan^4\theta + \tan^2\theta$$
$$= \tan^2\theta(1 + \tan^2\theta)$$
$$= \tan^2\theta \times \sec^2\theta \quad \mathbf{1}$$
$$= (\sec^2\theta - 1)\sec^2\theta$$
$$= \sec^4\theta - \sec^2\theta = \text{R.H.S.}$$
$\because$ L.H.S = R.H.S. **Hence Proved. 1**

Q. 3. Express the trigonometric ratio of sec *A* and tan *A* in terms of sin *A*. U [Board Term-1, 2015]

Sol.
$$\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1-\sin^2 A}} \quad \mathbf{1}$$
and
$$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1-\sin^2 A}} \quad \mathbf{1}$$
[CBSE Marking Scheme, 2015]

Q. 4. Prove that:
$$\frac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta} = 1$$
A [Board Term-1, 2015]

Sol.
$$\text{LHS} = \frac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta}$$
$$= \frac{(\sin^2\theta)^2 + (\cos^2\theta)^2}{1 - 2\sin^2\theta\cos^2\theta}$$
$$= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\theta\cos^2\theta}$$
$$= \frac{1 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\theta\cos^2\theta} = 1 = \text{RHS} \quad \mathbf{2}$$
Hence Proved

[CBSE Marking Scheme, 2015]

COMMONLY MADE ERROR

- Some students make mistakes to prove the sum and become confused.

ANSWERING TIP

- Follow step by step simplification to avoid errors.

Short Answer Type Questions-II — 3 marks each

AI **Q. 1.** $\sin\theta + \cos\theta = \sqrt{3}$**, then prove that** $\tan\theta + \cot\theta = 1$

A [CBSE SQP 2020]
[CBSE Delhi Set-I, 2020]

Sol. $\sin\theta + \cos\theta = \sqrt{3} \Rightarrow (\sin\theta + \cos\theta)^2 = 3$ **1**

$\Rightarrow 1 + 2\sin\theta\cos\theta = 3 \Rightarrow \sin\theta\cos\theta = 1$ **1**

$\therefore \tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 1$ **1**

[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

Given $\sin\theta + \cos\theta = \sqrt{3}$

Squaring on both sides,
$$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2 \quad \mathbf{1}$$
$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$
$$1 + 2\sin\theta\cos\theta = 3$$
$$2\sin\theta\cos\theta = 2$$
$$\sin\theta\cos\theta = 1 \quad \text{... (i) } \mathbf{1}$$
$$\therefore \tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$
$$= \frac{1}{\cos\theta\sin\theta}$$
$$= \frac{1}{1} = 1 \quad \text{[From equation (i)] } \mathbf{1}$$
Hence Proved

AI **Q. 2. Prove that:**
$$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0.$$
A [CBSE Delhi Set-II, 2020]

Sol. L.H.S.
$$= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$
$$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1 \quad \tfrac{1}{2}$$

$= 2\,[(\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta]$
$- 3\,(\sin^4\theta + \cos^4\theta) + 1$ ½
$[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$= 2\,(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta)$
$- 3\,(\sin^4\theta + \cos^4\theta) + 1\ [\because \sin^2\theta + \cos^2\theta = 1]$ **1**

$= -\sin^4\theta - \cos^4\theta - 2\sin^2\theta\cos^2\theta + 1$

$= -(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta) + 1$

$= -(\sin^2\theta + \cos^2\theta)^2 + 1$
$[\because (a+b)^2 = a^2 + b^2 + 2ab]$ ½

$= -1 + 1$

$= 0 = $ R.H.S. **Hence Proved.** ½

AI **Q. 3. Prove that: $\dfrac{\cot\theta + \text{cosec}\,\theta - 1}{\cot\theta - \text{cosec}\,\theta + 1} = \dfrac{1+\cos\theta}{\sin\theta}$.**

A [CBSE Delhi Set-III, 2020]

Sol. L.H.S. $= \dfrac{\cot\theta + \text{cosec}\,\theta - 1}{\cot\theta - \text{cosec}\,\theta + 1}$

$= \dfrac{\dfrac{\cos\theta}{\sin\theta} + \dfrac{1}{\sin\theta} - 1}{\dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta} + 1}$ ½

$= \dfrac{\sin\theta(\cos\theta - \sin\theta + 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$ ½

$= \dfrac{\sin\theta\cos\theta - \sin^2\theta + \sin\theta}{\sin\theta(\cos\theta + \sin\theta - 1)}$

$= \dfrac{\sin\theta\cos\theta + \sin\theta - (1-\cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$ **1**

$= \dfrac{\sin\theta(\cos\theta + 1) - [(1-\cos\theta)(1+\cos\theta)]}{\sin\theta(\cos\theta + \sin\theta - 1)}$ ½

$= \dfrac{(1+\cos\theta)(\sin\theta - 1 + \cos\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$

$= \dfrac{1+\cos\theta}{\sin\theta}$

= R.H.S. **Hence Proved.** ½

AI **Q. 4. If $\sin\theta + \cos\theta = \sqrt{2}$, prove that $\tan\theta + \cot\theta = 2$.**

A [CBSE OD Set-I, 2020]
[CBSE SQP, 2017] [Board Term-I, 2015]

Sol. Given, $\sin\theta + \cos\theta = \sqrt{2}$

On squaring both the sides, we get

$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 2$

$\Rightarrow 2\sin\theta\cos\theta = 2 - 1 = 1$

$\Rightarrow \dfrac{1}{\sin\theta\,\cos\theta} = 2$ ...(i) **1**

Now, $\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$\dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\,\sin\theta} = \dfrac{1}{\cos\theta\,\sin\theta}$ ...(ii) **1**

From (i) and (ii) we get

$\tan\theta + \cot\theta = 2$ **1**

AI **Q. 5. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or $\dfrac{1}{2}$.**

A [CBSE OD Set-II, 2020]

Sol. Given, $1 + \sin^2\theta = 3\sin\theta\cos\theta$

On dividing by $\sin^2\theta$ on both sides, we get

$\dfrac{1}{\sin^2\theta} + 1 = 3\cot\theta$ **1**

$\left[\because \cot\theta = \dfrac{\cos\theta}{\sin\theta}\right]$

$\Rightarrow \text{cosec}^2\theta + 1 = 3\cot\theta$

$\Rightarrow 1 + \cot^2\theta + 1 = 3\cot\theta$

$\Rightarrow \cot^2\theta - 3\cot\theta + 2 = 0$ **1**

$\Rightarrow \cot^2\theta - 2\cot\theta - \cot\theta + 2 = 0$

$\Rightarrow \cot\theta(\cot\theta - 2) - 1(\cot\theta - 2) = 0$

$\Rightarrow (\cot\theta - 2)(\cot\theta - 1) = 0$

If $\cot\theta = 1$ or 2

Then, $\tan\theta = 1$ or $\dfrac{1}{2}$.

Hence proved. 1

Q. 6. Show that : $\dfrac{\cos^2(45°+\theta) + \cos^2(45°-\theta)}{\tan(60°+\theta)\tan(30°-\theta)} = 1$.

U [CBSE OD Set-III, 2020]

Sol. L.H.S. $\dfrac{\cos^2(45°+\theta) + \cos^2(45°-\theta)}{\tan(60°+\theta)\tan(30°-\theta)}$

$= \dfrac{\cos^2(45°+\theta) + \cos^2\{90° - (45°+\theta)\}}{\tan(60°+\theta)\cot\{90° - (30°-\theta)\}}$ **1**

$= \dfrac{\cos^2(45°+\theta) + \sin^2(45°+\theta)}{\tan(60°+\theta)\cot(60°+\theta)}$ **1**

$[\because \cos^2\theta + \sin^2\theta = 1$ and $\tan\theta = \dfrac{1}{\cot\theta}]$

$= \dfrac{1}{1}$

$= 1 = $ R.H.S. **Hence, Proved. 1**

AI **Q. 7. Prove that:**
$(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

A [CBSE Delhi Set-I, II, III, 2019]

Sol. LHS $= \sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \cos^2\theta$
$+ \sec^2\theta + 2\cos\theta\sec\theta$ **1**

$= (\sin^2\theta + \cos^2\theta) + \text{cosec}^2\theta + \sec^2\theta$
$+ \dfrac{2\sin\theta}{\sin\theta} + 2\dfrac{\cos\theta}{\cos\theta}$

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 + 2$ **1½**

$= 7 + \cot^2\theta + \tan^2\theta = $ RHS **Hence Proved** ½

[CBSE Marking Scheme, 2019]

Q. 8. If $\sec\theta = x + \dfrac{1}{4x}$, prove that $\sec\theta + \tan\theta = 2x$ or $\dfrac{1}{2x}$.

A [CBSE Delhi Set-III, 2019]

Sol. $\sec\theta = x + \dfrac{1}{4x}$

$\sec^2\theta = x^2 + \dfrac{1}{16x^2} + 2\cdot x\dfrac{1}{4x}$

$$1 + \tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1$$

$$\tan^2\theta = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$\tan^2\theta = x^2 + \frac{1}{16x^2} - 2 \cdot x \cdot \frac{1}{4x}$$

$$\tan^2\theta = \left(x - \frac{1}{4x}\right)^2 \quad \mathbf{1}$$

Taking square root on both sides

$$\tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

If $\tan\theta = x - \frac{1}{4x}$

Given, $\sec\theta = x + \frac{1}{4x}$

Now, $\tan\theta + \sec\theta = 2x$

If $\tan\theta = -\left(x - \frac{1}{4x}\right) = -x + \frac{1}{4x}$ **1**

Given, $\sec\theta = x + \frac{1}{4x}$

Now, $\sec\theta + \tan\theta = \frac{1}{4x} + \frac{1}{4x} = \frac{1}{2x}$ **1**

Hence Proved.

Q. 9. Prove that: cot θ – tan θ = $\frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

U [CBSE SQP, 2018]

Sol. LHS = cot θ – tan θ **1**

$$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} \quad ½$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} \quad \mathbf{1}$$

$$= \frac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta\cos\theta} \quad ½$$

$$= \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta} = \text{RHS}$$

Hence Proved

[CBSE Marking Scheme, 2018]

Q. 10. Prove that: sin θ (1 + tan θ) + cos θ (1 + cot θ) = sec θ + cosecθ

U [CBSE SQP, 2018]

Sol. LHS = sin θ(1 + tan θ) + cos θ(1 + cot θ) **1**

$$= \sin\theta\left(1 + \frac{\sin\theta}{\cos\theta}\right) + \cos\theta\left(1 + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \sin\theta\left(\frac{\cos\theta + \sin\theta}{\cos\theta}\right) + \cos\theta\left(\frac{\sin\theta + \cos\theta}{\sin\theta}\right) \quad \mathbf{1}$$

$$= (\cos\theta + \sin\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$= \frac{\cos\theta + \sin\theta}{\cos\theta\sin\theta} = \text{cosec}\,\theta + \sec\theta = \text{RHS} \quad \mathbf{1}$$

Hence Proved

[CBSE Marking Scheme, 2018]

AI **Q. 11. Prove that:** $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2.$

U [CBSE Board Term-1, 2015]

Sol. LHS $= \frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$

$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)}$$

$$+ \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)} \quad \mathbf{1}$$

= (1 – sin θ cos θ) + (1 + sin θ cos θ) **1**

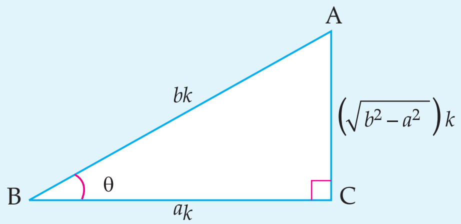
= 2 – sin θ cos θ + sin θ cos θ **1**

= 2 = RHS **Hence proved.**

Q. 12. If *b*cos θ = *a*, then prove that cosec θ + cot θ = $\sqrt{\frac{b+a}{b-a}}$.

U [CBSE Board Term-1, 2015]

Sol.



Given, $\cos\theta = \frac{a}{b}$

$AC^2 = AB^2 - BC^2$

$AC = \sqrt{b^2 - a^2}k$

$\text{cosec}\,\theta = \frac{b}{\sqrt{b^2 - a^2}}$, $\cot\theta = \frac{a}{\sqrt{b^2 - a^2}}$

$$\text{cosec}\,\theta + \cot\theta = \frac{b+a}{\sqrt{b^2 - a^2}} = \sqrt{\frac{b+a}{b-a}} \quad \mathbf{3}$$

[CBSE Marking Scheme, 2015]

Q. 13. Prove that: (cot θ – cosec θ)^2 = $\frac{1 - \cos\theta}{1 + \cos\theta}$

U [CBSE Board Term-1, 2015]

Sol. LHS = (cot θ – cosec θ)^2

$$= \left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)^2$$

$$= \left(\frac{\cos\theta - 1}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} \qquad (\because \sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

= RHS **Hence proved.**

[CBSE Marking Scheme, 2015] 3

COMMONLY MADE ERROR

- Some students tried to prove the identity by getting

$$\cot\theta - \text{cosec}\,\theta = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

ANSWERING TIP

- Students must be advised not to change the form of a given identity.

Long Answer Type Questions — 5 marks each

AI **Q. 1. Prove that** $\dfrac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \dfrac{1}{\sec A - \tan A}$

A [CBSE Delhi Set-I, 2019]

Sol. LHS $= \dfrac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$

Dividing num. & deno. by cos A,

$$= \frac{\tan A - 1 + \sec A}{\tan A + 1 - \sec A} \quad 1$$

$$= \frac{\tan A - 1 + \sec A}{(\tan A - \sec A) + (\sec^2 A - \tan^2 A)} \quad 1$$

$$= \frac{\tan A - 1 + \sec A}{(\tan A - \sec A) + (1 - \sec A - \tan A)} \quad 2$$

$$= \frac{-1}{\tan A - \sec A} = \frac{1}{\sec A - \tan A} = \text{RHS} \quad 1$$

Hence Proved

[CBSE Marking Scheme, 2019]

Detailed Answer :

$$\text{LHS} = \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$$

$$= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \times \frac{1 + \sin A}{1 + \sin A} \quad 1$$

$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{\sin A + \cos A - 1 + \sin^2 A + \cos A \sin A - \sin A}$$

$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{-1 + \cos A + (1 - \cos^2 A) + \sin A \cos A} \quad 1$$

$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{\cos A - \cos^2 A + \sin A \cos A}$$

$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{\cos A(1 - \cos A + \sin A)} \quad 1$$

$$= \frac{1 + \sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A \quad 1$$

$$= \frac{(\sec A + \tan A)}{(\sec A - \tan A)} \times (\sec A - \tan A)$$

$$= \frac{\sec^2 A - \tan^2 A}{\sec A - \tan A}$$

$$= \frac{1}{\sec A - \tan A} = \text{RHS}$$ **Hence Proved. 1**

Q. 2. Prove that:

$$\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A} = \frac{1}{1 - 2\cos^2 A}$$

A [CBSE Delhi Set-II, 2019]

Sol.

$$\text{LHS} = \frac{\dfrac{\sin^2 A}{\cos^2 A}}{\dfrac{\sin^2 A}{\cos^2 A} - 1} + \frac{\dfrac{1}{\sin^2 A}}{\dfrac{1}{\cos^2 A} - \dfrac{1}{\sin^2 A}} \quad 1$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A} \quad 1$$

$$= \frac{1}{\sin^2 A - \cos^2 A} \quad 1\tfrac{1}{2}$$

$$= \frac{1}{1 - 2\cos^2 A}$$ **Hence Proved 1½**

[CBSE Marking Scheme, 2019]

AI **Q. 3. Prove that:**

$$\frac{\sin\theta}{\cot\theta + \text{cosec}\,\theta} = 2 + \frac{\sin\theta}{\cot\theta - \text{cosec}\,\theta}$$

A [CBSE OD, Set-I, 2019]

Sol. No sequence

$$= \frac{\sin\theta}{\text{cosec}\,\theta + \cot\theta} - \frac{\sin\theta}{\cot\theta - \text{cosec}\,\theta}$$

$$= \frac{\sin\theta}{\text{cosec}\,\theta + \cot\theta} + \frac{\sin\theta}{\text{cosec}\,\theta - \cot\theta}$$

$$= \frac{\sin\theta[\text{cosec}\,\theta - \cot\theta + \text{cosec}\,\theta + \cot\theta]}{\text{cosec}^2\theta - \cot^2\theta} \quad 1+1$$

$$= \frac{\sin\theta(2\text{cosec}\,\theta)}{1}$$

$$= 2 \quad 2$$

Hence $\dfrac{\sin\theta}{\text{cosec}\,\theta + \cot\theta} = 2 + \dfrac{\sin\theta}{\cot\theta - \text{cosec}\,\theta}$ **1**

[CBSE Marking Scheme, 2019]

Q. 4. Prove that:

$$\frac{1}{1+\sin^2\theta} + \frac{1}{1+\cos^2\theta} + \frac{1}{1+\sec^2\theta} + \frac{1}{1+\text{cosec}^2\theta} = 2$$

A [CBSE Delhi, Region, 2019]

Topper Answer, 2019

Sol. To prove: $\frac{1}{1+\sin^2\theta} + \frac{1}{1+\cos^2\theta} + \frac{1}{1+\sec^2\theta} + \frac{1}{1+\text{cosec}^2\theta} = 2.$

Taking from LHS,

= LHS

$= \frac{1}{1+\sin^2\theta} + \frac{1}{1+\cos^2\theta} + \frac{1}{1+\text{cosec}^2\theta} + \frac{1}{1+\text{cosec}^2\theta}$

$= \frac{1}{1+\sin^2\theta} + \frac{1}{1+\text{cosec}^2\theta} + \frac{1}{1+\cos^2\theta} + \frac{1}{1+\sec^2\theta}$ [Re-arranging]

$= \frac{1}{1+\sin^2\theta} + \frac{1}{1+\left(\frac{1}{\sin^2\theta}\right)} + \frac{1}{1+\cos^2\theta} + \frac{1}{1+\left(\frac{1}{\cos^2\theta}\right)}$ $\left[\text{cosec}\,\theta = \frac{1}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}\right]$

$= \frac{1}{1+\sin^2\theta} + \frac{1\times\sin^2\theta}{1+\sin^2\theta} + \frac{1}{1+\cos^2\theta} + \frac{1\times\cos^2\theta}{1+\cos^2\theta}$

$= \frac{1+\sin^2\theta}{1+\sin^2\theta} + \frac{1+\cos^2\theta}{1+\cos^2\theta}$

$= 1 + 1$

$= 2$

$=$ RHS

LHS = RHS

Hence, proved! **5**

Q. 5. Prove that: $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A.$

U [CBSE Delhi/OD, 2018] [Board Term-I, 2015]

Sol. LHS $= \dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$$= \frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A - 1)} \quad 1$$

$$= \frac{\sin A(1-2(1-\cos^2 A))}{\cos A(2\cos^2 A - 1)} \quad 1$$

$$= \tan A\frac{(2\cos^2 A - 1)}{(2\cos^2 A - 1)} \quad 1\tfrac{1}{2}$$

= tanA = RHS **Hence Proved 1½**

[CBSE Marking Scheme, 2018]

Detailed Solution:

Topper Answer, 2018

Sol. To prove: $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A.$

(4) Simplifying LHS:

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$$

$$= \frac{\sin A\,(1 - 2\sin^2 A)}{\cos A\,(2\cos^2 A - 1)}$$

$$= \frac{\sin A}{\cos A}\left[\frac{1 - (2\sin^2 A)}{2\cos^2 A - 1}\right]$$

$$= \frac{\sin A}{\cos A}\left[\frac{\sin^2 A + \cos^2 A - 2\sin^2 A}{2\cos^2 A - (\sin^2 A + \cos^2 A)}\right] \quad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{\sin A}{\cos A}\left[\frac{\cos^2 A - \sin^2 A}{\cos^2 A - \sin^2 A}\right]$$

$$= \frac{\sin A}{\cos A} \times 1$$

$$= \tan A. \quad \left[\because \frac{\sin A}{\cos A} = \tan A\right]$$

LHS = RHS

hence proved. **5**

COMMONLY MADE ERROR

- Some common errors observed were:
  (i) Working with both sides together.
  (ii) Skipping of necessary steps so as to get the answer.
  (iii) Some opened the LHS expression but failed to simplify and come to the RHS.

ANSWERING TIP

- Ensure that while proving identities students proceed with either LHS or RHS but must not work with both sides simultaneously.

Q. 6. If $\sec\theta + \tan\theta = p$, then find the value of $\operatorname{cosec}\theta$.

U [CBSE SQP-2018]

Sol. $\sec\theta + \tan\theta = p$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= 1 + \sin\theta = p\cos\theta$$

$$= p\sqrt{1 - \sin^2\theta} \qquad \mathbf{1}$$

$$(1 + \sin\theta)^2 = p^2(1 - \sin^2\theta) \qquad \tfrac{1}{2}$$

$$1 + \sin^2\theta + 2\sin\theta = p^2 - p^2\sin^2\theta \qquad \mathbf{1}$$

$$(1 + p^2)\sin^2\theta + 2\sin\theta + (1 - p^2) = 0$$

$$D = 4 - 4(1 + p^2)(1 - p^2)$$

$$= 4 - 4(1 - p^4) = 4p^4 \qquad \mathbf{1}$$

$$\sin\theta = \frac{-2 \pm \sqrt{4p^4}}{2(1 + p^2)} = \frac{-1 \pm p^2}{(1 + p^2)} \qquad \tfrac{1}{2}$$

$$= \frac{p^2 - 1}{p^2 + 1}, -1$$

$$\therefore \quad \operatorname{cosec}\theta = \frac{p^2 + 1}{p^2 - 1}, -1 \qquad \textbf{Hence Proved 1}$$

[CBSE Marking Scheme, 2018]

Alternative Method:

Given $\sec\theta + \tan\theta = p$ ...(i)

we know $\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$p(\sec\theta - \tan\theta) = 1$

$\therefore \quad \sec\theta - \tan\theta = \frac{1}{p}$ ...(ii)

Add (i) and (ii) we get

$$2\sec\theta = p + \frac{1}{p} = \frac{p^2 + 1}{p}$$

$$\sec\theta = \frac{p^2 + 1}{2p}$$

Subtract (ii) from (i)

we get $$2\tan\theta = p - \frac{1}{p} = \frac{p^2 - 1}{p}$$

$$\therefore \quad \tan\theta = \frac{p^2-1}{2p}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{\cos\theta}{\sin\theta}\cos\theta}$$

$$\text{cosec}\,\theta = \frac{\sec\theta}{\tan\theta}$$

$$= \frac{\frac{p^2+1}{2p}}{\frac{p^2-1}{2p}}$$

$$\therefore \quad \text{cosec}\,\theta = \frac{p^2+1}{p^2-1}.$$

Q. 7. Prove that: $\dfrac{\cos\theta-\sin\theta+1}{\cos\theta+\sin\theta-1} = \text{cosec}\,\theta+\cot\theta$

[CBSE SQP, 2017-18]

Sol. LHS $= \dfrac{\cos\theta-\sin\theta+1}{\cos\theta+\sin\theta-1}$

$$= \frac{\sin\theta(\cos\theta-\sin\theta+1)}{\sin\theta(\cos\theta+\sin\theta-1)} \quad \mathbf{1}$$

$$= \frac{\sin\theta\cos\theta-\sin^2\theta+\sin\theta}{\sin\theta(\cos\theta+\sin\theta-1)}$$

$$= \frac{\sin\theta\cos\theta+\sin\theta-(1-\cos^2\theta)}{\sin\theta(\cos\theta+\sin\theta-1)} \quad \mathbf{1}$$

$$= \frac{\sin\theta(\cos\theta+1)-[(1-\cos\theta)(1+\cos\theta)]}{\sin\theta(\cos\theta+\sin\theta-1)}$$

$$= \frac{(1+\cos\theta)(\sin\theta-1+\cos\theta)}{\sin\theta(\cos\theta+\sin\theta-1)} \quad \mathbf{1}$$

$$= \frac{(1+\cos\theta)(\cos\theta+\sin\theta-1)}{\sin\theta(\cos\theta+\sin\theta-1)} = \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta} \quad \mathbf{1}$$

$= \text{cosec}\,\theta + \cot\theta =$ RHS **Hence Proved 1**

Q. 8. If $\text{cosec}\,\theta + \cot\theta = p$**, then prove that**

$$\cos\theta = \frac{p^2-1}{p^2+1}.$$

U [Board Term-1, 2016]

Sol. RHS $= \dfrac{p^2-1}{p^2+1}$

$$= \frac{(\text{cosec}\,\theta+\cot\theta)^2-1}{(\text{cosec}\,\theta+\cot\theta)^2+1} \quad \mathbf{1}$$

$$= \frac{\text{cosec}^2\,\theta+\cot^2\,\theta+2\,\text{cosec}\,\theta\cot\theta-1}{\text{cosec}^2\,\theta+\cot^2\,\theta+2\,\text{cosec}\,\theta\cot\theta+1} \quad \mathbf{1}$$

$$= \frac{1+\cot^2\theta+\cot^2\theta+2\,\text{cosec}\,\theta\cot\theta-1}{\text{cosec}^2\,\theta+\text{cosec}^2\,\theta-1+2\,\text{cosec}\,\theta\cot\theta+1} \quad \mathbf{1}$$

$$= \frac{2\cot\theta(\cot\theta+\text{cosec}\,\theta)}{2\,\text{cosec}\,\theta(\text{cosec}\,\theta+\cot\theta)} \quad \mathbf{1}$$

$$= \frac{\cos\theta}{\sin\theta}\times\sin\theta$$

$= \cos\theta =$ LHS. **Hence proved. 1**

[CBSE Marking Scheme, 2016]

Alternative Method:

$\text{cosec}\,\theta + \cot\theta = p$ (given)

We know $\text{cosec}^2\theta - \cot^2\theta = 1$

$(\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta) = 1$

$p(\text{cosec}\,\theta - \cot\theta) = 1$

$\therefore \quad \text{cosec}\,\theta - \cot\theta = \dfrac{1}{p}$ ...(i)

$\text{cosec}\,\theta + \cot\theta = p$ ...(ii)

from eq. (i) and (ii)

$$\text{cosec}\,\theta = \frac{p^2+1}{2p}$$

and $$\cot\theta = \frac{p^2-1}{2p}$$

$$\cos\theta = \frac{\cos\theta}{\sin\theta}\times\sin\theta$$

$$= \cot\theta\times\frac{1}{\text{cosec}\,\theta}$$

$$= \frac{p^2-1}{2p}\times\frac{2p}{p^2+1}$$

$$\therefore \quad \cos\theta = \frac{p^2-1}{p^2+1}$$

Hence Proved

AI **Q. 9. If** $\sec\theta + \tan\theta = p$**, show that** $\sec\theta - \tan\theta = \dfrac{1}{p}$**.**

Hence, find the values of $\cos\theta$ **and** $\sin\theta$**.**

A [Board Term-1, 2015]

Sol. $$\frac{1}{p} = \frac{1}{\sec\theta+\tan\theta}\times\frac{(\sec\theta-\tan\theta)}{\sec\theta-\tan\theta} \quad \mathbf{1}$$

$$\frac{1}{p} = \frac{\sec\theta-\tan\theta}{\sec^2\theta-\tan^2\theta} = \sec\theta-\tan\theta \quad \mathbf{1}$$

Solving, $\sec\theta + \tan\theta = p$ and $\sec\theta - \tan\theta = \dfrac{1}{p}$

We get $$\sec\theta = \frac{1}{2}\left(p+\frac{1}{p}\right) = \frac{p^2+1}{2p} \quad \mathbf{1}$$

and $$\tan\theta = \frac{1}{2}\left(p-\frac{1}{p}\right) = \frac{p^2-1}{2p} \quad \mathbf{1}$$

$\therefore \quad \cos\theta = \dfrac{2p}{p^2+1}$ and $\sin\theta = \dfrac{p^2-1}{p^2+1}$ **1**

[CBSE Marking Scheme, 2015]

Q. 10. Prove that $b^2x^2 - a^2y^2 = a^2b^2$, if :
(i) $x = a \sec \theta$, $y = b \tan \theta$, or
(ii) $x = a \operatorname{cosec} \theta$, $y = b \cot \theta$.

U [Board Term-1, 2015]

Sol. (i) $\frac{x^2}{a^2} = \sec^2\theta, \frac{y^2}{b^2} = \tan^2\theta$ ½

or, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta = 1.$

$\therefore$ $b^2x^2 - a^2y^2 = a^2b^2.$ **Hence Proved. 2**

(ii) $\frac{x^2}{a^2} = \operatorname{cosec}^2\theta, \frac{y^2}{b^2} = \cot^2\theta$ ½

or, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\therefore$ $b^2x^2 - a^2y^2 = a^2b^2$ **Hence Proved. 2**

Q. 11. If $\operatorname{cosec}\theta - \cot\theta = \sqrt{2}\cot\theta$, then prove that $\operatorname{cosec}\theta + \cot\theta = \sqrt{2}\operatorname{cosec}\theta$.

U [Board Term-1, 2015]

Sol. Given, $\operatorname{cosec}\theta - \cot\theta = \sqrt{2}\cot\theta$

Squaring both the sides,

$\operatorname{cosec}^2\theta + \cot^2\theta - 2\operatorname{cosec}\theta\cot\theta = 2\cot^2\theta$

or, $\operatorname{cosec}^2\theta - \cot^2\theta = 2\operatorname{cosec}\theta\cot\theta$ 1½

$[\because a^2 - b^2 = (a+b)(a-b)]$

or, $(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 2\operatorname{cosec}\theta\cot\theta$

Given : $(\operatorname{cosec}\theta - \cot\theta = \sqrt{2}\cot\theta)$ 1

or, $\operatorname{cosec}\theta + \cot\theta = \frac{2\operatorname{cosec}\theta\cot\theta}{\sqrt{2}\cot\theta}$ 1

$\operatorname{cosec}\theta + \cot\theta = \sqrt{2}\operatorname{cosec}\theta$ 1½

Hence Proved.

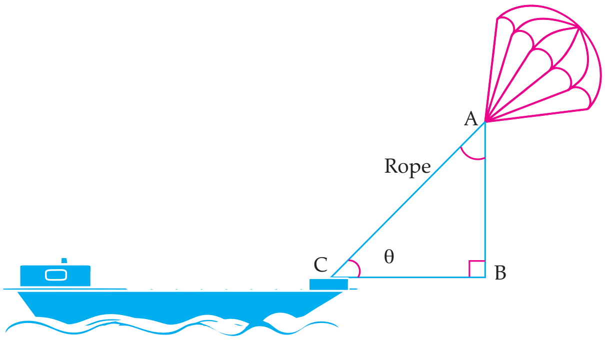
[CBSE Marking Scheme, 2015]

Visual Case Based Questions — 4 marks each

Note: Attempt any four sub parts from each question. Each sub part carries 1 mark

Q. 1. 'Skysails' is that genre of engineering science that uses extensive utilization of wind energy to move a vessel in the sea water. The sky sails technology allows the towing kite to gain a height of anything between 100 m to 300 m. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a telescopic mast that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing answer the questions: C + AE



(i) In the given figure, if $\tan\theta = \cot(30° + \theta)$, where θ and $30° + \theta$ are acute angles, then the value of θ is:

(a) 45° (b) 30°
(c) 60° (d) None of these

Sol. Correct option: **(b)**.

Explanation: Given, $\tan\theta = \cot(30° + \theta)$

$= \tan[90° - (30° + \theta)]$

$= \tan(90° - 30° - \theta)$

$\Rightarrow \tan\theta = \tan(60° - \theta)$

$\Rightarrow \theta = 60° - \theta$

$\Rightarrow 2\theta = 60°$

$\Rightarrow \theta = 30°.$ 1

(ii) The value of tan 30°. cot 60° is:

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$
(c) 1 (d) $\frac{1}{3}$

Sol. Correct option: **(d)**.

Explanation:

$\tan 30° \times \cot 60° = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$

$= \frac{1}{3}.$ 1

(iii) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ and be at a vertical height of 200 m ?

(a) 400 m (b) 300 m
(c) 100 m (d) 200 m

Sol. Correct option: **(a)**.

Explanation: In ΔABC, we have

$\theta = 30°, AB = 200$ m

Then, $\sin 30° = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC}$

$\Rightarrow \frac{1}{2} = \frac{200}{AC}$

$\Rightarrow AC = 400$ m. 1

(iv) If $\cos A = \frac{1}{2}$, then the value of $9\cot^2 A - 1$ is :

(a) 1 (b) 3
(c) 2 (d) 4

Sol. Correct option: **(c)**.

Explanation: Given, $\cos A = \frac{1}{2}$

$$\Rightarrow \quad \cos A = \cos 60°$$
$$\Rightarrow \quad A = 60°$$

Then, $9 \cot^2 A - 1 = 9(\cot 60°)^2 - 1$

$$= 9\left(\frac{1}{\sqrt{3}}\right)^2 - 1$$

$$= 9 \times \frac{1}{3} - 1 = 3 - 1$$

$$= 2$$ **1**

(v) In the given figure, the value of (sin C + cos A) is:

(a) 1 **(b) 2**

(c) 3 **(d) 4**

Sol. Correct option: **(a)**.

Explanation: We have,

$$AB = 200 \text{ m and } AC = 400 \text{ m}$$

[Proved in Q.3]

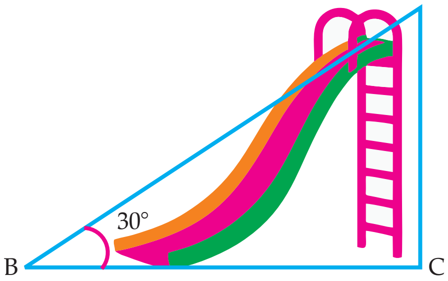
Then, $\sin C + \cos A = \frac{AB}{AC} + \frac{AB}{AC}$

$$= 2 \times \frac{AB}{AC}$$

$$= 2 \times \frac{200}{400} = 1$$ **1**

Q. 2. Authority wants to construct a slide in a city park for children. The slide was to be constructed for children below the age of 12 years. Authority prefers the top of the slide at a height of 4 m above the ground and inclined at an angle of 30° to the ground.

Based on the following figure related to the slide answer the questions: C + AE



(i) The distance of AB is :

(a) 8 m **(b) 6 m**

(c) 5 m **(d) 10 m**

Sol. Correct option: **(a)**.

Explanation: We have, $\angle B = 30°$ and

$$AC = 4 \text{ m}$$

Then, $\sin 30° = \frac{AC}{AB}$

$$\Rightarrow \quad \frac{1}{2} = \frac{4}{AB}$$

$$\Rightarrow \quad AB = 8 \text{ m.}$$ **1**

(ii) The value of $\sin^2 30° + \cos^2 60°$ is:

(a) $\frac{1}{4}$ **(b) $\frac{1}{2}$**

(c) $\frac{3}{4}$ **(d) $\frac{3}{2}$**

Sol. Correct option: **(b)**.

Explanation:

$$\sin^2 30° + \cos^2 60° = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{2}{4} = \frac{1}{2}.$$ **1**

(iii) If $\cos A = \frac{1}{2}$, then the value of $12 \cot^2 A - 2$ is:

(a) 5 **(b) 4**

(c) 3 **(d) 2**

Sol. Correct option: **(d)**.

Explanation: since, $\cos A = \frac{1}{2}$

$$\Rightarrow \quad \cos A = \cos 60°$$
$$\Rightarrow \quad A = 60°$$

Then $12 \cot^2 A - 2 = 12(\cot 60°) - 2$

$$= 12\left(\frac{1}{\sqrt{3}}\right)^2 - 2$$

$$= 12 \times \frac{1}{3} - 2$$

$$= 4 - 2 = 2.$$ **1**

(iv) In the given figure, the value of (sin C × cos A) is:

(a) $\frac{1}{3}$ **(b) $\frac{1}{2}$**

(c) $\frac{1}{4}$ **(d) $\frac{1}{5}$**

Sol. Correct option: **(b)**.

Explanation: Since, $AC \perp BC$,

then $\angle C = 90°$

$$\sin C \times \cos A = \sin 90° \times \frac{AC}{AB}$$

$$= 1 \times \frac{4}{8}$$

$$= \frac{1}{2}$$ **1**

(v) In the given figure, if $AB + BC = 25$ cm and $AC = 5$ cm, then the value of BC is:

(a) 25 cm **(b) 15 cm**

(c) 10 cm **(d) 12 cm**

Sol. Correct option: **(d)**.

Explanation: We have, $\angle C = 90°$

$$AB + BC = 25 \text{ cm and } AC = 5 \text{ cm}$$

Let BC be x cm, then $AB = (25 - x)$ cm

By using Pythagoras theorem,

$$AB^2 = BC^2 + AC^2$$

$$\Rightarrow \quad (25 - x^2) = x^2 + (5)^2$$

$$\Rightarrow \quad 625 - 50x + x^2 = x^2 + 25$$

$$\Rightarrow \quad 50x = 600$$

$$\Rightarrow \quad x = \frac{600}{50} = 12$$

Hence, $BC = 12$ cm. **1**